A Method for the Direct Synthesis of Cascaded Quintuplets

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ABSTRACT — A new lowpass prototype configuration for a cascaded quintuplet is presented. The prototype can be converted into the cascaded quintuplet format using a series of equivalent circuit transformations. The method permits the direct synthesis of an arbitrary number of in-line cascaded quintuplets from the filter impedance function.

I. INTRODUCTION

The cascaded quintuplet is a cross-coupled filter structure that is used in microwave filter topologies to realize three finite frequency transmission zeros [1,2]. The coupling arrangement for a cascaded quintuplet is shown in Fig. 1.

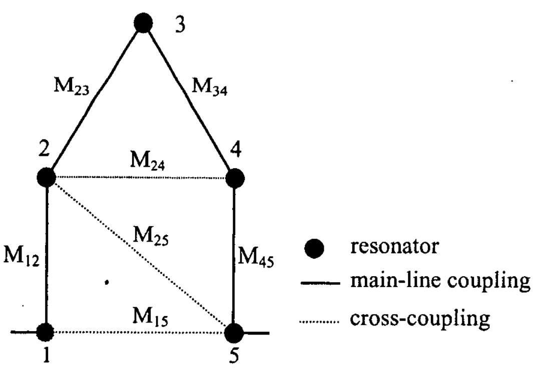

Fig. 1 Cascaded Quintuplet

Historically, the body of published work on the subject of cascaded quintuplet synthesis has been quite limited. Cameron [3] described the method of turning which can be used to extract a single cascaded quintuplet from the filter admittance function. Liang and Zhang [1], as well as Yildrim et al. [2] have each described a methodology for constructing cascaded quintuplet filters using cascaded quadruplet (CQ) and cascaded triplet (CT) building blocks. However, neither paper presented explicit design formulae for the synthesis of the cascaded quintuplet.

The cascaded quintuplet lowpass prototype presented in this paper will also be constructed from lower order transmission zero-producing filter networks. The two structures that are used to construct the cascaded quintuplet are the tri-section [4] and the general section [5] lowpass prototypes.

II. DIRECT SYSNTHESIS OF A QUINTUPLET

In Fig. 1, each of the resonator nodes consists of a shunt capacitor and shunt frequency invariant susceptance connected to ground. In a similar fashion to the method described by Levy [6], the resonator elements at nodes 1 and 5 are pre-extracted and post extracted with respect to the partial quintuplet. The resulting partial cascaded quintuplet is shown in Fig. 2.

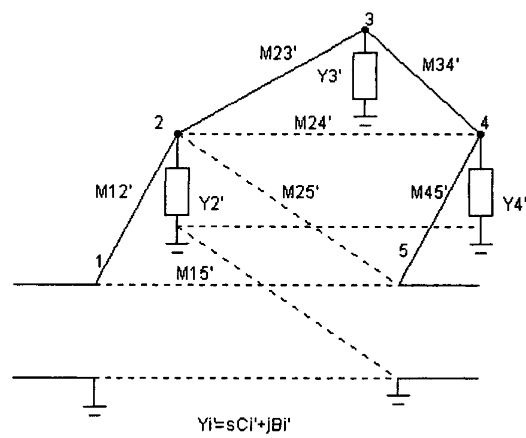

Fig. 2 Partial Cascaded Quintuplet

Using the methodology described by Scanlon and Rhodes [7] and Levy [6], the network transfer function of the cascaded quintuplet can be extracted from the overall network transfer function of the filter. The network transfer function can then be transformed to the admittance matrix equivalent.

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The general form of the partial admittance matrix is given by

$$[Y]=\begin{bmatrix} Y_{11} & Y_{15} \\ Y_{51} & Y_{55} \end{bmatrix} \quad (1).$$

Because the network is symmetric, lossless and reciprocal, the admittances Y_{15} and Y_{51} are identical frequency dependent functions that describe the quintuplet's transimmittance behavior. Y_{15} and Y_{51} are also third order polynomials in frequency ω, and there are therefore three frequencies ω_1, ω_2, and ω_3 at which there is no transmission between nodes 1 and 5. This characteristic equation for the cascaded quintuplet is given by

$$a_3\omega^3 + a_2\omega^2 + a_1\omega + a_0 = 0 \quad (2)$$

where

$$a_0 = M_{15}\left(B_2B_3B_4 + M_{24}^2B_3 - B_4 + 2M_{34}M_{24}\right) - 1 \quad (3)$$

$$a_1 = M_{15}\left(C_2\left(M_{34}^2 - B_3B_4\right) - B_2\left(C_3B_4 - B_3C_4\right) - \left(M_{24}^2C_3 - C_4\right)\right) \quad (4)$$

$$a_2 = M_{15}\left(C_2C_3B_4 + C_2B_3C_4 + B_2C_3C_4\right)) \quad (5)$$

$$a_3 = M_{15}C_2C_3C_4 \quad (6).$$

It should be noted that there are a number of different solutions to this equation. The combinations include:

- three finite frequency poles, i.e. ω_1, ω_2 and ω_3,
- a complex frequency pole pair that is symmetric about the real axis and one finite frequency pole, i.e. $\omega_1 \pm j\sigma_1$, ω_3 (or)
- a real frequency pole pair and one finite frequency pole i.e. $\pm j\sigma_1$ and ω_3.

The solutions listed above are obtained by defining the susceptance term, Y_i, as $Y_i = j((\omega - j\sigma)C_i + B_i)$.

III. NETWORK TRANSFORMATION

Since the Cascaded Quintuplet uniquely exists as a sub-network of the overall filter network, a cascaded quintuplet can be extracted separately from the rest of the filter network [5,6]. Rather than writing a special synthesis program, it is more convenient to transform cascaded lowpass filters of defined topology into the cascaded quintuplet format using a number of circuit transformation steps. The lowpass prototype network for a complete cascaded quintuplet is shown in Fig. 3.

The tenth degree filter shown in Figure 4a will be used as a design example. This filter has a fourth ordered attenuation pole at infinity and a pair of third order finite frequency poles, giving a total filter degree of 10.

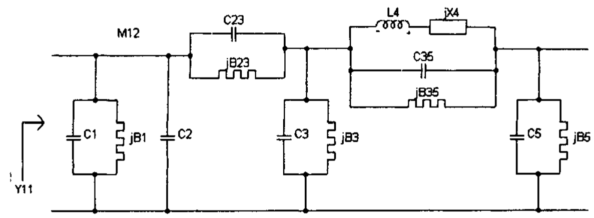

Fig. 3 Low pass prototype network of a quintuplet

Following the procedure outlined by Levy [6], each series inductor and frequency invariant reactance is replaced by a shunt capacitor and frequency invariant susceptance interposed between a pair of admittance inverters. The non-unity terminating resistance is incorporated into one of the inverters at this stage, yielding the circuit in Fig. 4b.

From inspection, the admittance matrix of the circuit section between nodes 1 and 5 of Fig. 4b is given by

$$[Y]=\begin{bmatrix} Y_{11} & -jM_{12} & 0 & 0 & 0 \\ -jM_{12} & Y_{22} & Y_{23} & 0 & 0 \\ 0 & Y_{23} & Y_{33} & -j & Y_{35} \\ 0 & 0 & -j & Y_{44} & j \\ 0 & 0 & Y_{35} & j & Y_{55} \end{bmatrix} \quad (7)$$

where

$$Y_{11} = sC_1 + jB_1;\quad Y_{22} = s(C_2 + C_{23}) + jB_{23};$$
$$Y_{33} = s(C_3 + C_{23} + C_{35}) + j(B_3 + B_{23} + B_{35});$$
$$Y_{44} = sC_4 + jB_4$$
$$Y_{55} = s(C_5 + C_{35}) + j(B_5 + B_{35});\quad Y_{23} = -sC_{23} - jB_{23};$$
$$Y_{35} = -sC_{35} - jB_{35} \quad (8).$$

In order to convert this circuit into cascaded quintuplet format, a series of matrix manipulations are employed in order to eliminate the frequency dependence of the non-diagonal elements in (7). The first matrix operation that will be performed will result in the annihilation of the frequency dependent couplings in Y_{35} and Y_{53}. This is achieved by multiplying row 3 and column 3 by x_0 and adding this result to row 5 and column 5 respectively, where x_0 is given by

$$x_0 = \frac{C_{35}}{(C_3 + C_{23} + C_{35})} \quad (9).$$

As a result of this operation, two frequency dependent admittances are created, Y_{25} and Y_{52}. In addition, undesired admittances Y_{53} and Y_{35} are retained. These undesired admittance terms will be eliminated in subsequent steps.

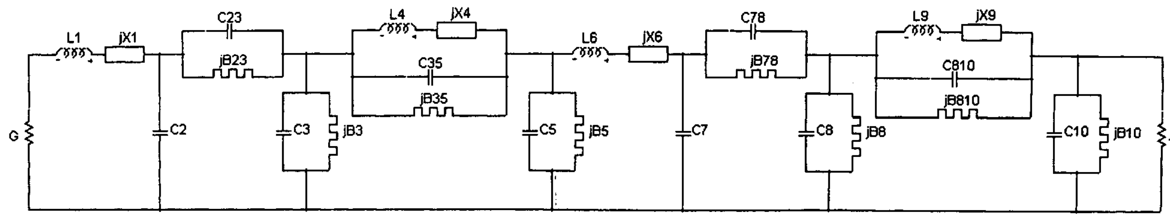

Fig. 4a Prototype network prior to transformation

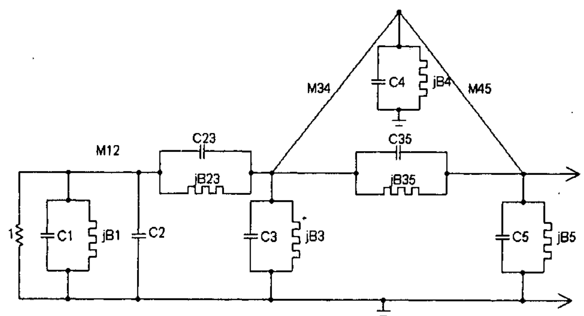

Fig. 4b Partially transformed network (first quintuplet shown)

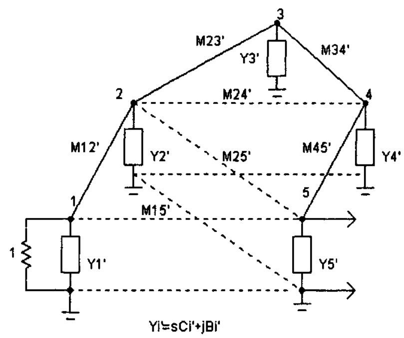

Fig. 4c Transformed network (first quintuplet shown)

The next matrix operation will eliminate the frequency dependency of admittances Y_{32} and Y_{23}. This is achieved by multiplying row 3 and column 3 by x_1 and adding this result to row 2 and column 2 respectively. The multiplying factor x_1 is given by

$$x_1 = \frac{C_{23}}{(C_3 + C_{23} + C_{35})} \qquad (10).$$

The next matrix reduction operation eliminates the frequency dependency of admittances Y_{25} and Y_{52}. This is accomplished by multiplying row 2 and column 2 by x_2 and adding this result to row 5 and column 5 respectively, where

$$x_2 = \frac{C_{23}C_{35}}{C_2(C_3 + C_{23} + C_{35}) + C_{23}(C_3 + C_{35})} \qquad (11).$$

At this point, all the non-diagonal admittances have been transformed into frequency invariant terms. In addition, the only undesired admittances remaining are Y_{35} and Y_{53}. In order to eliminate these two admittances, row 4 and column 4 are multiplied by x_3 and this result is added to row 3 and column 3 respectively. The multiplying factor, x_3, is given by

$$x_3 = -\frac{Y_{35}}{Y_{45}} \qquad (12)$$

and

$$Y_{35} = -jB_{23}\frac{C_{23}C_{35}}{[C_2(C_3 + C_{23} + C_{35}) + C_{23}(C_3 + C_{35})]} + j\frac{(B_3 + B_{23} + B_{35})C_{23}}{(C_3 + C_{23} + C_{35})}\frac{C_{23}C_{35}}{[C_2(C_3 + C_{23} + C_{35}) + C_{23}(C_3 + C_{35})]} - jB_{35} + j\frac{(B_3 + B_{23} + B_{35})C_{35}}{(C_3 + C_{23} + C_{35})}$$

$$Y_{45} = j\frac{C_2(C_3 + C_{23}) + C_3C_{23}}{C_2(C_3 + C_{23} + C_{35}) + C_{23}(C_3 + C_{35})} \qquad (13).$$

Resulting from this operation, the admittances Y_{34} and Y_{43} now have frequency dependent admittance terms. The final matrix operation eliminates the frequency dependence of these two admittances. In order to accomplish this, row 3 and column 3 are multiplied by x_4 and the result added to row 4 and column 4 respectively, where x_4 is given by

$$x_4 = -\frac{C_4x_3}{C_3 + C_{23} + C_{35} + x_3^2C_4} \qquad (14).$$

The admittance matrix is now in the pre-normalized cascaded quintuplet format. The admittance matrix has the form

$$[Y] = \begin{bmatrix} Y_{11} & -jM_{12} & 0 & 0 & jM_{15} \\ -jM_{12} & Y_{22} & jM_{23} & jM_{24} & jM_{25} \\ 0 & jM_{23} & Y_{33} & jM_{34} & 0 \\ 0 & jM_{24} & jM_{34} & Y_{44} & jM_{45} \\ jM_{15} & jM_{25} & 0 & jM_{45} & Y_{55} \end{bmatrix} \qquad (15).$$

where

$$M_{23} = -B_{23} + \frac{(B_3+B_{23}+B_{35})C_{23}}{(C_3+C_{23}+C_{35})} - \frac{x_3 C_{23}}{(C_3+C_{23}+C_{35})}$$

$$M_{34} = -1 + x_3 B_4 + x_4 B_{33}$$

$$M_{15} = -\frac{M_{12} C_{23} C_{35}}{C_2(C_3+C_{23}+C_{35}) + C_{23}(C_3+C_{35})}$$

$$M_{24} = -\frac{C_{23}}{(C_3+C_{23}+C_{35})} + x_4 M_{23}$$

$$M_{25} = -\frac{(B_{23}C_{35} + B_{35}C_{23})}{(C_3+C_{23}+C_{35})} + \frac{C_{23}C_{35}(B_3+B_{23}+B_{35})}{(C_3+C_{23}+C_{35})^2} +$$

$$+ \frac{C_{23}C_{35}B_{22}}{C_2(C_3+C_{23}+C_{35}) + C_{23}(C_3+C_{35})}$$

$$Y_{11} = sC_{11} + jB_{11}$$

$$Y_{22} = sC_{22} + jB_{22}$$

$$C_{22} = C_2 + \frac{C_{23}(C_3+C_{35})}{(C_3+C_{23}+C_{35})};$$

$$B_{22} = B_{23} + \left(\frac{(B_3+B_{23}+B_{35})C_{23}}{(C_3+C_{23}+C_{35})}\right)\frac{C_{23}}{(C_3+C_{23}+C_{35})}$$

$$-\frac{2B_{23}C_{23}}{(C_3+C_{23}+C_{35})}$$

$$Y_{33} = sC_{33} + jB_{33}$$

$$C_{33} = C_3 + C_{23} + C_{35} + x_3^2 C_4$$

$$B_{33} = B_3 + B_{23} + B_{35} + x_3^2 B_4 - 2x_3$$

$$Y_{44} = sC_{44} + jB_{44}$$

$$C_{44} = C_4 + x_3 x_4 C_4$$

$$B_{44} = B_4 + x_3 x_4 B_4 - x_4 + x_4 M_{34}$$

$$Y_{55} = sC_{55} + jB_{55} \qquad (16)$$

$$C_{55} = -\frac{(C_{23}C_{35})^2}{(C_3+C_{23}+C_{35})[C_2(C_3+C_{23}+C_{35}) + C_{23}(C_3+C_{35})]}$$

$$+ C_5 + C_{35} - \frac{C_{35}^2}{(C_3+C_{23}+C_{35})}$$

$$B_{55} = B_5 + B_{35} - \frac{2B_{35}C_{35}}{(C_3+C_{23}+C_{35})} +$$

$$\left(\frac{(B_3+B_{23}+B_{35})C_{23}}{(C_3+C_{23}+C_{35})}\right)\frac{C_{35}}{(C_3+C_{23}+C_{35})}$$

$$-\frac{2B_{23}(C_{35})^2 C_{23}}{(C_3+C_{23}+C_{35})(C_2(C_3+C_{23}+C_{35}) + C_{23}(C_3+C_{35}))}$$

$$-\frac{2B_{35}(C_{23})^2 C_{35}}{(C_3+C_{23}+C_{35})(C_2(C_3+C_{23}+C_{35}) + C_{23}(C_3+C_{35}))}$$

$$+\frac{2(B_3+B_{23}+B_{35})(C_{35})^2 C_{23}}{(C_3+C_{23}+C_{35})^2(C_2(C_3+C_{23}+C_{35}) + C_{23}(C_3+C_{35}))}$$

$$+ B_{22}\left[\frac{C_{23}C_{35}}{(C_2(C_3+C_{23}+C_{35}) + C_{23}(C_3+C_{35}))}\right]^2$$

At this stage in the synthesis process, all of the main-line admittance inverters generally have non-unity magnitudes, and their polarities may be either positive or negative. In a manner similar to that described by Levy [6], all the main-line admittance inverters can be normalized to unity except for coupling M_{34}. This involves applying a sequence of normalization operations based on the well known admittance matrix scaling operation on row i column i. To have the desired effect, the scaling factor w is selected as

$$w = \frac{1}{M_{i-1,i}} \qquad (17).$$

V. CONCLUSIONS

A new lowpass prototype along with a new direct synthesis method for cascaded quintuplets has been presented. The extraction technique has no limitation with respect to filter order nor the number of cascaded quintuplets that can be extracted. The technique removes previous limitations of network order and greatly simplifies the synthesis of lower order networks containing multiple cascaded quintuplets.

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